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INVESTIGATION OF ADVANCED TURBULENCE MODELING APPROACHES FOR AEROACOUSTIC PROBLEMS

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Abstract. The influence of a synthetic method for high frequency turbulence on aeroacoustic simulations with turbulence models is investigated. The sound pressure level (SPL) of a detached eddy simulation with and without the synthetic method are compared for a benchmark test case, The results show that the synthetic method is able to increase the simulation accuracy of the high frequency spectrum.

1 INTRODUCTION

Aeroacoustic studies have drawn increasingly more attention in the past years. One of the main tasks is to accurately simulate the acoustic field with a reasonable computational cost. An efficient approach to simulate the acoustic field generated by low Mach number flows is the expansion about incompressible flow (EIF). This approach decomposes the compressible field into an incompressible field and acoustic fluctuations [4, 8]. Based on this decomposition, the acoustic field is governed by the linearized Euler equations (LEE) with an acoustic source term, which can be obtained by solving the incompressible Navier-Stokes equations [6]. The simulation accuracy of the acoustic field strongly depends on that of the flow field. For the simulation of turbulent flow in engineering problems, direct numerical simulation (DNS) cannot be applied due to its unaffordable computational cost. A turbulence model is usually adopted to characterize the unresolved turbulence scales, leading to a significant reduction in computational cost. In Large Eddy Simulation (LES), about 10% of the turbulence is modeled while about 90% is resolved [10]. Delayed Detached Eddy Simulation (DDES) is a hybrid RANS/LES method, wich switches between LES and Reynolds Averaged Navier-Stokes (RANS) modes according to the grid resolution [11, 12]. However, its main drawback is the loss of high-frequency components in the flow. Consequently, the high-frequency acoustic quantities can not be calculated accurately.

In this work, we study the influence of different turbulence models on the simulation accuracy of acoustic quantities. Considering an aeroacoustic benchmark test case, we compare the accuracy loss of acoustic quantities for different turbulence models including LES and DDES. To compensate the accuracy loss of acoustic quantities in the high-frequency region, the high frequency fluctuation is rebuilt using a synthetic reconstruction model. We adopt this model for the DDES simulation and investigate the performance change. A clear improvement of the high-frequency spectrum can be observed.

2 GOVERNING EQUATIONS

2.1 Linearized Euler equations for aeroacoustic simulation

The Expansion about Incompressible Flow method (EIF) assumes that the compressible flow field at low Mach number can be decomposed into an incompressible flow field and an acoustic field [8, 9]

$$u_i = u_i^{\mathrm{inc}} + u_i^{\mathrm{ac}}, \tag{1}$$

$$p = p^{\mathrm{inc}} + p^{\mathrm{ac}}, \tag{2}$$

$$\rho = \rho^{\mathrm{inc}} + \rho^{\mathrm{ac}}, \tag{3}$$

where u_i, p and ρ are the velocity, pressure and density of compressible flow and superscripts inc and ac represent the components of incompressible flow and acoustic field respectively [6].

The unsteady incompressible flow is governed by the incompressible Navier-Stokes equations

$$\frac{\partial u_i^{\mathrm{inc}}}{\partial x_i} = 0, \tag{4}$$

$$\rho^{\mathrm{inc}} \frac{\partial \left(u_i^{\mathrm{inc}}\right)}{\partial t} + \rho^{\mathrm{inc}} \frac{\partial \left(u_i^{\mathrm{inc}} u_j^{\mathrm{inc}}\right)}{\partial x_j} = \frac{\partial \tau_i^{\mathrm{inc}}}{\partial x_j} - \frac{\partial p^{\mathrm{inc}}}{\partial x_i} + \rho^{\mathrm{inc}} f_i, \tag{5}$$

with time t, external body force f_i and shear stress τ_i^{inc}. For newtonian fluid, the shear stress is given by

$$\tau_i^{\mathrm{inc}} = \mu \left(\frac{\partial u_i^{\mathrm{inc}}}{\partial x_j} + \frac{\partial u_j^{\mathrm{inc}}}{\partial x_i} \right) \tag{6}$$

The governing equations for the acoustic quantities, which is called the linearized Euler equations (LEE), are given as

$$\frac{\partial \rho^{\mathrm{ac}}}{\partial t} + \rho^{\mathrm{inc}} \frac{\partial u_i^{\mathrm{ac}}}{\partial x_i} + u_i^{\mathrm{inc}} \frac{\partial \rho^{\mathrm{ac}}}{\partial x_i} = 0, \tag{7}$$

$$\rho^{\mathrm{inc}} \frac{\partial u_i^{\mathrm{ac}}}{\partial t} + \rho^{\mathrm{inc}} u_j^{\mathrm{inc}} \frac{\partial u_i^{\mathrm{ac}}}{\partial x_j} + \frac{\partial p^{\mathrm{ac}}}{\partial x_i} = 0, \tag{8}$$

$$\frac{\partial p^{\mathrm{ac}}}{\partial t} + c^2 \rho^{\mathrm{inc}} \frac{\partial u_i^{\mathrm{ac}}}{\partial x_i} + c^2 u_i^{\mathrm{inc}} \frac{\partial \rho^{\mathrm{ac}}}{\partial x_i} = -\frac{\partial p^*}{\partial t}, \tag{9}$$

where p^* is the sum of the incompressible pressure and the synthetic pressure fluctuation. More details will be found in Chap. 5.

2.2 Detached Eddy Simulation model

The $\zeta - f$ model is used as background RANS model for the DDES model. The equations are given as [3]

$$\frac{\partial k}{\partial t} + \overline{u_j^{\mathrm{inc}}} \frac{\partial k}{\partial x_j} = P_k + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] - \varepsilon, \tag{10}$$

$$\frac{\partial \varepsilon}{\partial t} + \overline{u_j^{\mathrm{inc}}} \frac{\partial \varepsilon}{\partial x_j} = \frac{C_{\varepsilon 1} P_k - C_{\varepsilon 2}\varepsilon}{\tau} + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial \varepsilon}{\partial x_j}\right], \tag{11}$$

$$\frac{\partial \zeta}{\partial t} + \overline{u_j^{\mathrm{inc}}} \frac{\partial \zeta}{\partial x_j} = f - \frac{\zeta}{k} P_k + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial \zeta}{\partial x_j}\right], \tag{12}$$

$$L^2 \nabla f - f = \frac{1}{\tau}\left(C_1 + C_2' \frac{P_k}{\varepsilon}\right)\left(\zeta - \frac{2}{3}\right), \tag{13}$$

where ε is the dissipation, f is the elliptic relaxation term, L is the length scale and τ is the time scale of turbulence.

The DDES model is a hybrid LES/RANS model, which switches between URANS and LES according to the numerical resolution [12]. The dissipation term ε in the k equation is modified to

$$\varepsilon = \frac{k^{3/2}}{l_{\mathrm{turb}}}, \tag{14}$$

where l_{turb} is the length scale of DDES

$$l_{\mathrm{turb}} = l_{\mathrm{RANS}} - f_d \max(0, d - C_{\mathrm{DES}} \Delta\phi). \tag{15}$$

3 IMPLEMENTATION OF SYNTHETIC METHOD

3.1 Synthetic reconstruction of high frequency turbulence

The synthetic method reconstructs the turbulent velocity fluctuations from a given dissipation rate and set of second moments [1]. The velocity fluctuations are given as

$$u_i^{\mathrm{syn}}(x_j, t) = a_{ik} \sqrt{\frac{2}{N}} \sum_{n=1}^{N} \left[p_k^n \cos\left(\hat{d}_j^n \hat{x}_j^n + \omega^n \hat{t}\right) + q_k^n \sin\left(\hat{d}_j^n \hat{x}_j^n + \omega^n \hat{t}\right)\right] \tag{16}$$

$$\hat{x}_j = 2\pi x_j / L, \quad \hat{t} = 2\pi t/\tau, \quad \hat{d}_j^n = d_j^n \frac{V}{c^n}, \quad V = L/\tau, \tag{17}$$

$$c^n = \sqrt{\frac{3}{2}\overline{u_l' u_m'} d_l^n d_k^n / d_k^n d_k^n}, \quad p_i^n = \epsilon_{ijk} \eta_j^n d_k^n, \quad q_i^n = \epsilon_{ijk} \xi_j^n d_k^n, \tag{18}$$

$$\eta_i^n, \xi_i^n = N(0,1), \quad \omega^n = N(1,1), \quad d_i^n = N(0, \frac{1}{2}). \tag{19}$$

where a_{ij} is the Cholesky decomposition of $\overline{u_i' u_j'}$, L and τ are local length and time scales, and N is set to 100. η_i^n, ξ_i^n, ω^n and d_i^n are random numbers with given mean and variance. After the velocity fluctuations are calculated, the pressure fluctuations are obtained using a pressure correction method, which is the standard method in the flow solver FASTEST which is used to solve the incompressible Navier-Stokes equations [7].

3.2 Numerical realization of flow solver and acoustic solver

The work flow of the aeroacoustic simulation after new implementation is shown in Fig.1.

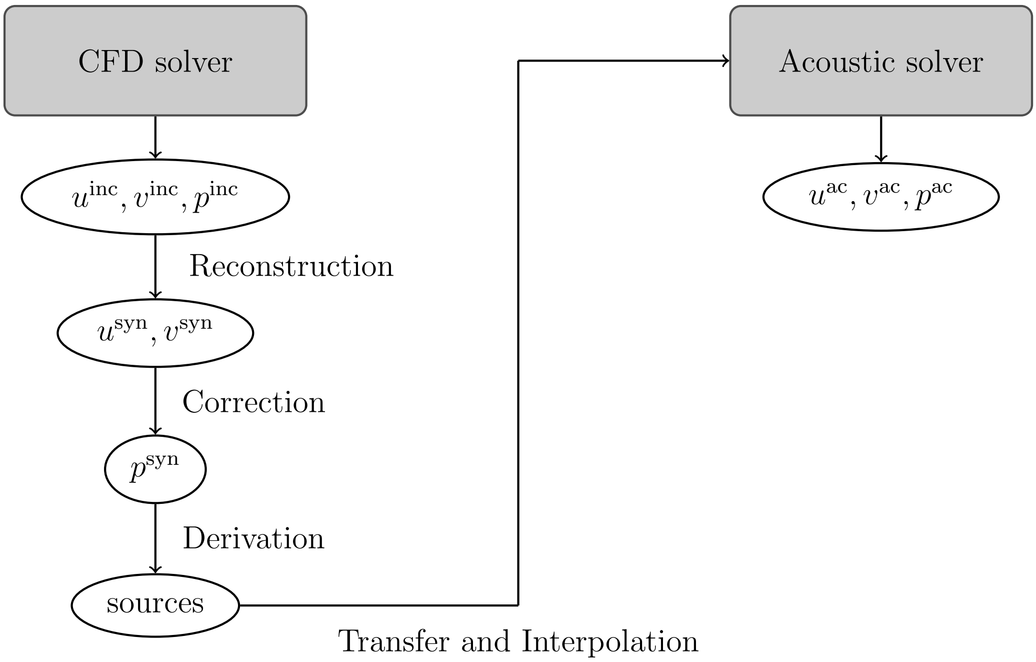


Figure 1: Numerical realization of flow solver and acoustic solver.

First, we solve for the incompressible flow quantities on a flow grid. Second, we reconstruct the high frequency turbulence using the synthetic method. Then we calculate the synthetic pressure using the pressure-correction method. Thereafter, the acoustic source term is calculated by differentiating the pressure with respect to time. Then, we transfer and interpolate the acoustic sources onto an acoustic grid, which has a larger range and a coarse resolution. For the coupling of these two grids, a trilinear interpolation is utilized. Finally, we solve the LEE to calculate the acoustic quantities.

4 SIMULATION

The acoustic quantities generated by the turbulent flow around a circular cylinder is measured by Etkin et al [2], which is used as reference data in this work. The Mach number is 0.2 and the Reynolds number is approximately 60000. A sketch of the flow domain is given in Fig.2, where D is 0.0125m. The z-direction has a length of $4D$, which is sufficient to capture the three dimensional turbulent features. The experimental data are collected at a point, which is above the cylinder and at a distance of 48D from the cylinder's center.

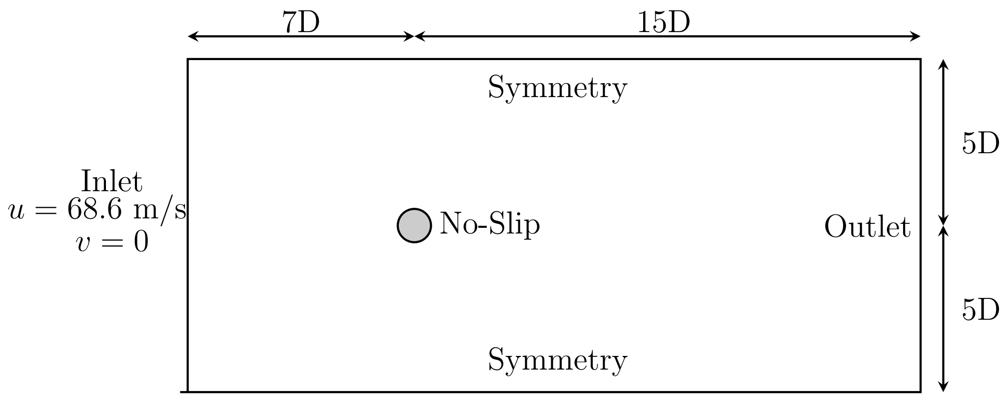


Figure 2: Sketch and boundary conditions of flow domain.

30000 time steps are observed, which is sufficient to acquire a fully developed von Kármán vortex street and a stable propagation process of acoustics. For the LES a flow grid with about 2.5 million cells is used, resulting in a simulation time of approximately 4 days. The grid for the DDES model is created according to the grid resolution requirements from [13]. The DDES model needs 2.5 days to finish the simulation. For all cases, the time step of the flow is set to 3×10^{-6}s, which ensures that the sampling frequency of different simulations are the same.

5 RESULTS AND DISCUSSION

The SPL of different turbulence models without synthetic method are compared with the experimental data in Fig.3. With less computational cost, the DDES method gives a comparable simulation result to that of LES. With regard of the high frequency spectrum, both DDES and LES simulations deviate from the experimental data. The reason is that the high frequency turbulence is filtered or averaged in the turbulence models, so that the acoustic solver can not calculate the high frequency spectrum correctly [5].

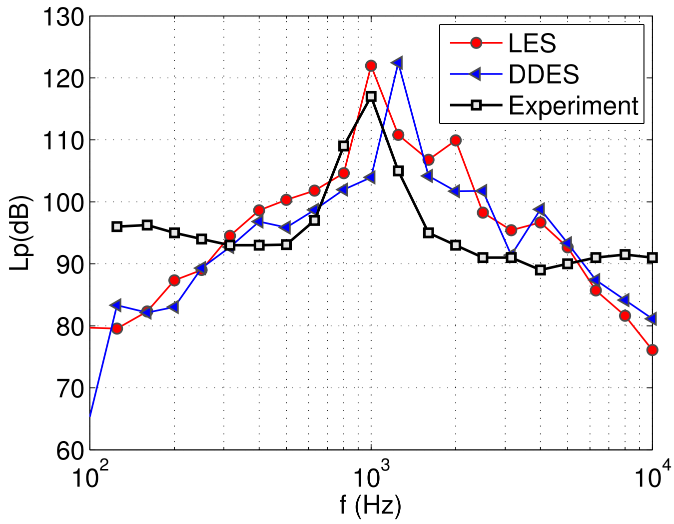


Figure 3: Comparison of SPL of different turbulence models with experimental data [5]

With the help of the synthetic method, the high frequency turbulence is reconstructed. In order to avoid stability problems, a coefficient is used in the pressure. The pressure is given as

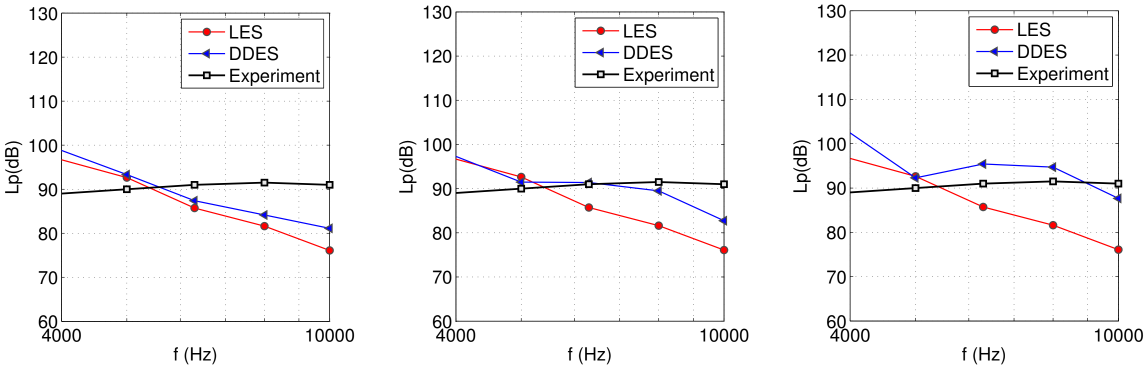
$$p^* = \alpha(p^{\text{syn}} + p^{\text{inc}}) + (1 - \alpha)p^{\text{inc}}, \tag{20}$$

where α is the coefficient, which can control the amount of synthetic pressure that is actually used in the simulation, p^{syn} is the synthetic pressure and p^{inc} is the incompressible pressure.

Figure 4 illustrates the simulation results with the synthetic method. The DDES results with the synthetic method shows an obvious improvement of the high frequency spectrum in comparison to that without the synthetic method. The coefficient α also has an influence on the simulation result. Specifically, when $\alpha = 0.1$, the simulation result reaches the best agreement with the experimental data. When α increases further, the simulation result deviates from the experimental data. It is shown that the DDES simulation with the synthetic method achieves even more accurate results than the LES simulation in terms of the high frequency spectrum, even though the DDES simulation requires less cells and less computational cost.

6 CONCLUSION

The synthetic method has been implemented and investigated in the context of aeroacoustic simulations. It has been shown that the synthetic method is able to improve the DDES simulation results in terms of the high frequency spectrum. The coefficient α for pressure is calibrated to 0.1. The DDES simulation provides more accurate results than the LES simulation, which makes the DDES method a very promising technique, since it requires less computational cost.



(a) SPL of turubulence models without synthetic method

(b) SPL of turubulence models with synthetic method, $\alpha = 0.1$

(c) SPL of turubulence models with synthetic method, $\alpha = 0.2$

Figure 4: Comparison of SPL of different turbulence models with and without synthetic method

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